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The Effects of Piston and Tilt Errors on
the Performance of Multiple Mirror Telescopes

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The degradation due to random segment jitter and phase mismatch on the average far field, on axis irradiance of a laser beam transmitted from a telescope with a segmented primary mirror is computed. The derived expression is exact within the stated assumptions and offers a more accurate alternative, for this application, to the commonly used Strehl approximation.

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For a system designed to transmit a laser beam to a receiver or target, there are four options available to increase the intensity of the deposited radiation:

1. Use a shorter wavelength laser.
2. Minimize system errors.
3. Increase laser power.
4. Increase the size of the transmitting optics.

Each evolutionary step taken in advancing the technology usually incorporates improvements in at least the last three areas. Although building a bigger transmitting telescope might appear to be one of the most straightforward approaches to improving system performance, for applications involving very long propagation distances, it will become increasingly difficult to fabricate and control optics of the required size.

An approach which would potentially alleviate some of the problems inherent in simply scaling up a transmitting telescope of conventional design is to divide the laser beam up among several smaller telescopes or to make the primary mirror segmented. The advantages of this approach to telescope design for applications in high resolution imagery is widely appreciated. The Multiple Mirror Telescope currently in operation on Mt Hopkins collects light with six hexagonal primary mirrors. Proposed plans for a larger follow-on to the Space Telescope include a segmented primary.

In either case, the individual telescopes or segments would have to be phased together so that the pieces of the laser beam would add coherently on the target. In addition to the phasing requirement, the different subapertures of the transmitter would all have to be pointed to the same spot on the target.

In practice, these requirements would be satisfied only imperfectly. There would be some residual phasing error and some pointing jitter of the mirror segments which would vary from one segment to the next. The effect of these residual errors on the far field, on axis irradiance is computed below.

Consider a transmitting aperture of area A divided into N circular subapertures of area $A/N = \pi D^2/4$. If monochromatic radiation of wavelength λ is transmitted through the N subapertures and brought to a focus a distance z from the telescope, then the field in the focal plane is given by (neglecting purely phase terms)[1]

$$U(\vec{r}) \cong (1/\lambda z) \int_A U(\vec{r}_1) \exp(-ik\vec{r}\cdot\vec{r}_1/z)\, d^2\vec{r}_1$$

$$= (1/\lambda z) \sum_{n=1}^{N} \int_{A_n} U_n(\vec{r}_1) \exp(-ik\vec{r}\cdot\vec{r}_1/z)\, d^2\vec{r}_1 \qquad (1)$$

where $k = 2\pi/\lambda$, and U_n is the field in the n^{th} subaperture A_n. We assume that each U_n is a uniform amplitude wave with a phase consisting of random piston and tilt.

$$U_n(\vec{r}_1) = (P/A)^{1/2} \exp\left\{ik\, [a_n + b_n\, (x_1 - \bar{x}_n) + c_n\, (y_1 - \bar{y}_n)]\right\} \qquad (2)$$

where $(\bar{x}_n, \bar{y}_n)$ is the centroid of the $n^{\underline{th}}$ subaperture, and P is the total power in the beam. The irradiance pattern in the focal plane is then given by

$$I(\vec{r}) = U(\vec{r})\, U^*(\vec{r})$$

$$= (P/\lambda^2 z^2 A)\left\{\sum_{n,m} \int_{A_n} d^2\vec{r_1} \int_{A_m} d^2\vec{r_2} \exp[-ik\vec{r}\cdot(\vec{r_1}-\vec{r_2})/z]\right.$$

$$\times \exp\{ik[a_n + b_n(x_1-\bar{x}_n) + c_n(y_1-\bar{y}_n)]\} \tag{3}$$

$$\left.\times \exp\{-ik[a_m + b_m(x_2-\bar{x}_m) + c_m(y_2-\bar{y}_m)]\}\right\}$$

We assume that a_n, b_n, and c_n are pairwise independent Gaussian random variables with mean 0 and variances given by

$$\begin{aligned} &< a_n^2 > = \sigma_o^2 \\ &< b_n^2 > = < c_n^2 > = \sigma_T^2 \end{aligned} \tag{4}$$

$<\text{-}>$ denotes an ensemble average. The average focal plane irradiance distribution is

$$< I(\vec{r}) > = (P/\lambda^2 z^2 A)\left\{ \sum_n \iint_{A_n} d^2\vec{r_1}\, d^2\vec{r_2} \exp[-ik\vec{r}\cdot(\vec{r_1}-\vec{r_2})/z]\right.$$

$$\times < \exp[ikb_n(x_1-x_2)] > < \exp[ikc_n(y_1-y_2)] >$$

$$+ \sum_{n\neq m} \int_{A_n} d^2\vec{r_1} \int_{A_m} d^2\vec{r_2} \exp[-ik\vec{r}\cdot(\vec{r_1}-\vec{r_2})/z]$$

$$\times < \exp(ika_n) > < \exp(-ika_m) > < \exp[ikb_n(x_1-\bar{x}_n)] >$$

$$\left.\times < \exp[-ikb_m(x_2-\bar{x}_m)] > < \exp[ikc_n(y_1-\bar{y}_n)] > < \exp[-ikc_m(y_2-\bar{y}_m)] >\right\}$$

$$= (P/\lambda^2 z^2 A) \left\{ \sum_n \iint_{A_n} d^2\vec{r}_1 \, d^2\vec{r}_2 \, \exp[-ik\vec{r}\cdot(\vec{r}_1-\vec{r}_2)/z] \right.$$

$$\times \exp\{-k^2 \sigma_T^2 [(x_1-x_2)^2 + (y_1-y_2)^2]/2\}$$

$$+ \sum_{n\neq m} \int_{A_n} d^2\vec{r}_1 \int_{A_m} d^2\vec{r}_2 \, \exp[-ik\vec{r}\cdot(\vec{r}_1-\vec{r}_2)/z] \tag{5}$$

$$\left. \times \exp(-k^2\sigma_o^2) \exp\{-k^2\sigma_T^2[(x_1-\bar{x}_n)^2 + (y_1-\bar{y}_n)^2 + (x_2-\bar{x}_m)^2 + (y_2-\bar{y}_m)^2]/2\} \right\}$$

Transforming to sum and difference coordinates in the first set of double integrals and translating the origin to the subaperture centroids in the second set of integrals in equation (5), it is straightforward to show that

$$< I(\vec{r}) > = (P/\lambda^2 z^2 A) \left\{ 4AD^2 \int_0^1 \rho d\rho \, \exp(-k^2\sigma_T^2 D^2\rho^2/2) \right.$$

$$\times [\cos^{-1}(\rho) - \rho\sqrt{1-\rho^2}] \, J_o(kDr\rho/z)$$

$$+ 4\pi^2 \exp(-k^2\sigma_o^2) \sum_{n\neq m} \exp[-ik\vec{r}\cdot(\bar{r}_n-\bar{r}_m)] \tag{6}$$

$$\left. \times [\int_0^{D/2} r_1 dr_1 \, \exp(-k^2\sigma_T^2 r_1^2/2) \, J_o(krr_1/z)]^2 \right\}$$

Evaluating $< I(\vec{r}) >$ on axis, we obtain

$$< I(0) > = (P/\lambda^2 z^2 A) \left\{ 4AD^2 \int_0^1 \rho d\rho \right.$$

$$\times \exp(-k^2\sigma_T^2 D^2\rho^2/2) \, [\cos^{-1}(\rho) - \rho\sqrt{1-\rho^2}] \tag{7}$$

$$\left. + 4\pi^2 \exp(-k^2\sigma_o^2)(N^2-N) \, [\int_0^{D/2} r_1 \, \exp(-k^2\sigma_T^2 r_1^2/2) dr_1]^2 \right\}$$

The integrations appearing in equation (7) can be performed analytically. The first integral can be evaluated by integrating by parts and then using a trigonometric substitution to transform it to a standard integral identity for Bessel functions. The result is

$$\begin{aligned} < I(0) > = (PA/\lambda^2 z^2) \Big\{ & [8/(Nk^2\sigma_T^2 D^2)] \\ & \times \Big[1 - \exp(-k^2\sigma_T^2 D^2/4) \, [I_0(k^2\sigma_T^2 D^2/4) + I_1(k^2\sigma_T^2 D^2/4)]\Big] \\ & + [64/(N^2k^4\sigma_T^4 D^4)] \, (N^2-N) \exp(-k^2\sigma_o^2) \\ & \times [1 - \exp(-k^2\sigma_T^2 D^2/8)]^2 \Big\} \end{aligned} \qquad (8)$$

I_0 and I_1 denote the modified Bessel functions of order 0 and 1 respectively.[2]

Let $p_o = \sigma_o/\lambda$ and $p = \sigma_T D/\lambda$. Then p_o is the rms piston error in waves and p is the rms tilt error measured in units of λ/D, the diffraction angle of the subapertures. If we normalize $< I(0) >$ by its diffraction limited value, $PA/\lambda^2 z^2$, we obtain the Strehl ratio which we denote by I_{rel}. As a function of p_o and p, I_{rel} is given by

$$\begin{aligned} I_{rel} = {} & (2/N\pi^2p^2) \{1 - \exp(-\pi^2p^2)[I_0(\pi^2p^2) + I_1(\pi^2p^2)]\} \\ & + 4 \, [(N-1)/(N\pi^4p^4)] \exp(-4\pi^2p_o^2) \, [1 - \exp(-\pi^2p^2/2)]^2 \end{aligned} \qquad (9)$$

Equation (9) can be compared to the Strehl approximation:

$$I_{rel} \cong \exp[-4\pi^2(N-1)p_o^2/N - \pi^2p^2/2] \tag{10}$$

The Strehl approximation gives lower values than equation (9), but it becomes more accurate as the number of subapertures increases. For $N \geqq 6$, the Strehl approximation is within a few percent of the correct value for $I_{rel} \geqq 0.4$.

It is instructive to note the behavior of I_{rel} for certain special cases.

Case 1. $N = 1$. As expected, I_{rel} is independent of p_o when there is only one aperture.

$$I_{rel} = (2/\pi^2p^2) \left| 1 - \exp(-\pi^2p^2)[I_0(\pi^2p^2) + I_1(\pi^2p^2)] \right| \tag{11}$$

Case 2. $\sigma_T = 0$. The limit as $p \to 0$ can be evaluated by substituting the power series for the exponential function and the Bessel functions in equation (9).

$$I_{rel} = [1 + (N-1) \exp(-4\pi^2p_o^2)]/N \tag{12}$$

for $p = 0$. Note that $I_{rel} \to 1$ as $p_o \to 0$, and $I_{rel} \to 1/N$ as $p_o \to \infty$.

Case 3. $N \to \infty$. For large N,

$$I_{rel} = 4 \exp(-4\pi^2p_o^2) [1- \exp(-\pi^2p^2/^2)]^2/(\pi^4p^4) \tag{13}$$

Care should be exercised in interpreting equation (13). If A is held constant as $N \to \infty$, then $D \to 0$. Then for a fixed value of p, $\sigma_T \to \infty$. In applying equation (13), it is necessary to keep in mind what is varying and what is being held constant.

If we let $p \to 0$ in equation (13), then

$$I_{rel} \to \exp(-4\pi^2 p_o{}^2) \tag{14}$$

Equation (14) is simply the usual Strehl approximation which is exact in the limit of phase aberrations with zero correlation length. That is just the situation represented by uncorrelated piston errors over N subapertures when $N \to \infty$.

Figures 1-5 show the dependence of I_{rel} on p_o and p for different values of N. In Figure 1, I_{rel} is shown as a function of p for the case N = 1. There is no dependence on p_o, and the simplified expression given in equation (11) holds.

Figures 2, 3, and 4, show results for the cases N = 6, 18, and 36, respectively. (These values correspond to hexagonal arrays of circular apertures with a hole in the center.) In each figure, I_{rel} is plotted as a function of p. The different curves in each figure correspond to different values of p_o. p_o is varied from 0.0 to 0.5 in increments of 0.05; i.e., the piston error is incremented by 1/20 wave. Of course, I_{rel} decreases as p_o increases so the top curve in each figure corresponds to $p_o = 0$, and each successively lower curve corresponds to the next higher value of p_o.

Figure 5 corresponds to the limit $N \to \infty$. The curves were drawn using equation (13). The piston error is again varied in increments of 1/20 wave.

Although many important sources of aberrations have been ignored and some simplifying assumptions have been used, equation (9) should be useful for determining the phasing and pointing accuracy requirements for the component pieces of a multiple mirror or segmented mirror telescope.

REFERENCES

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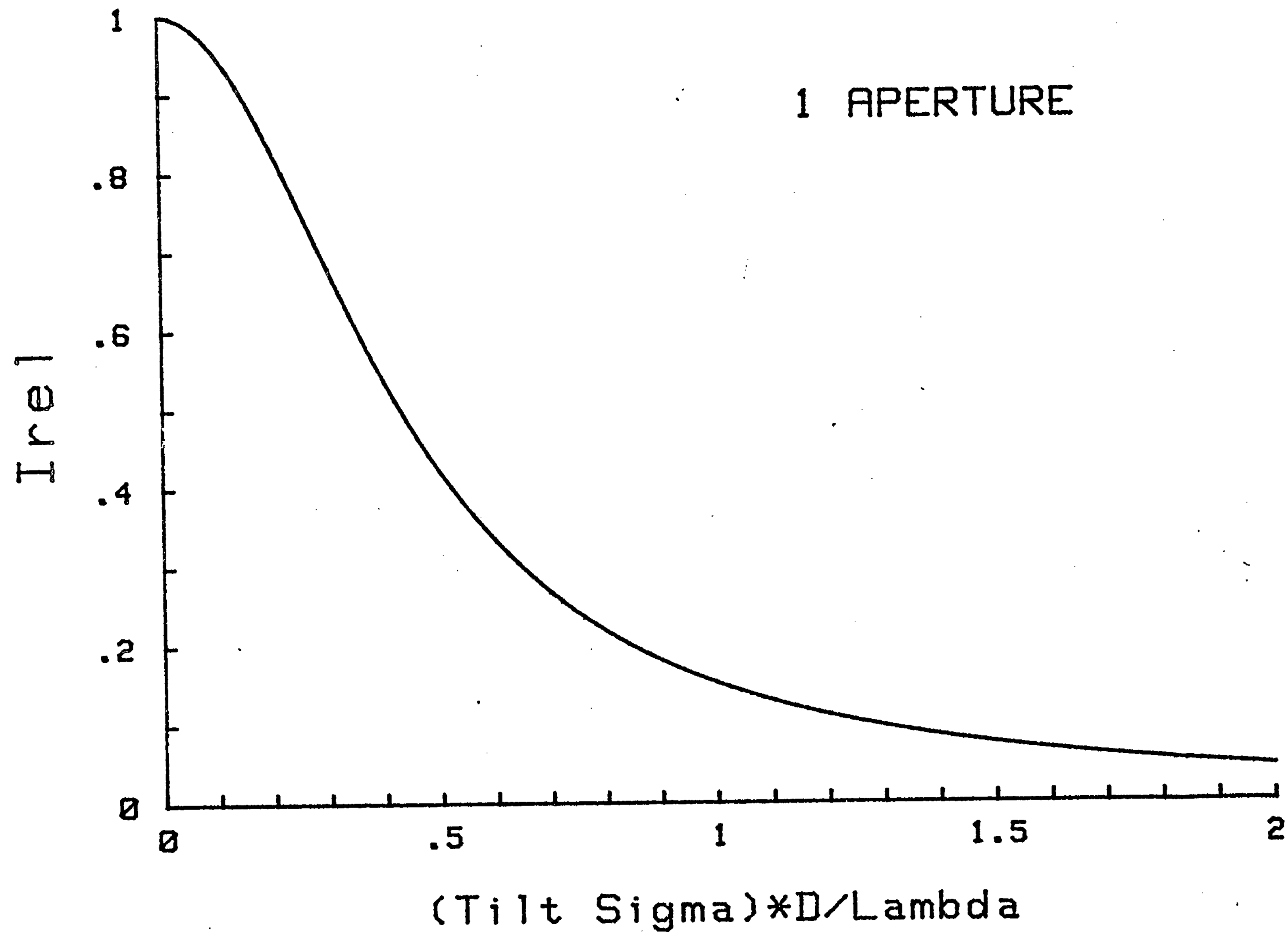
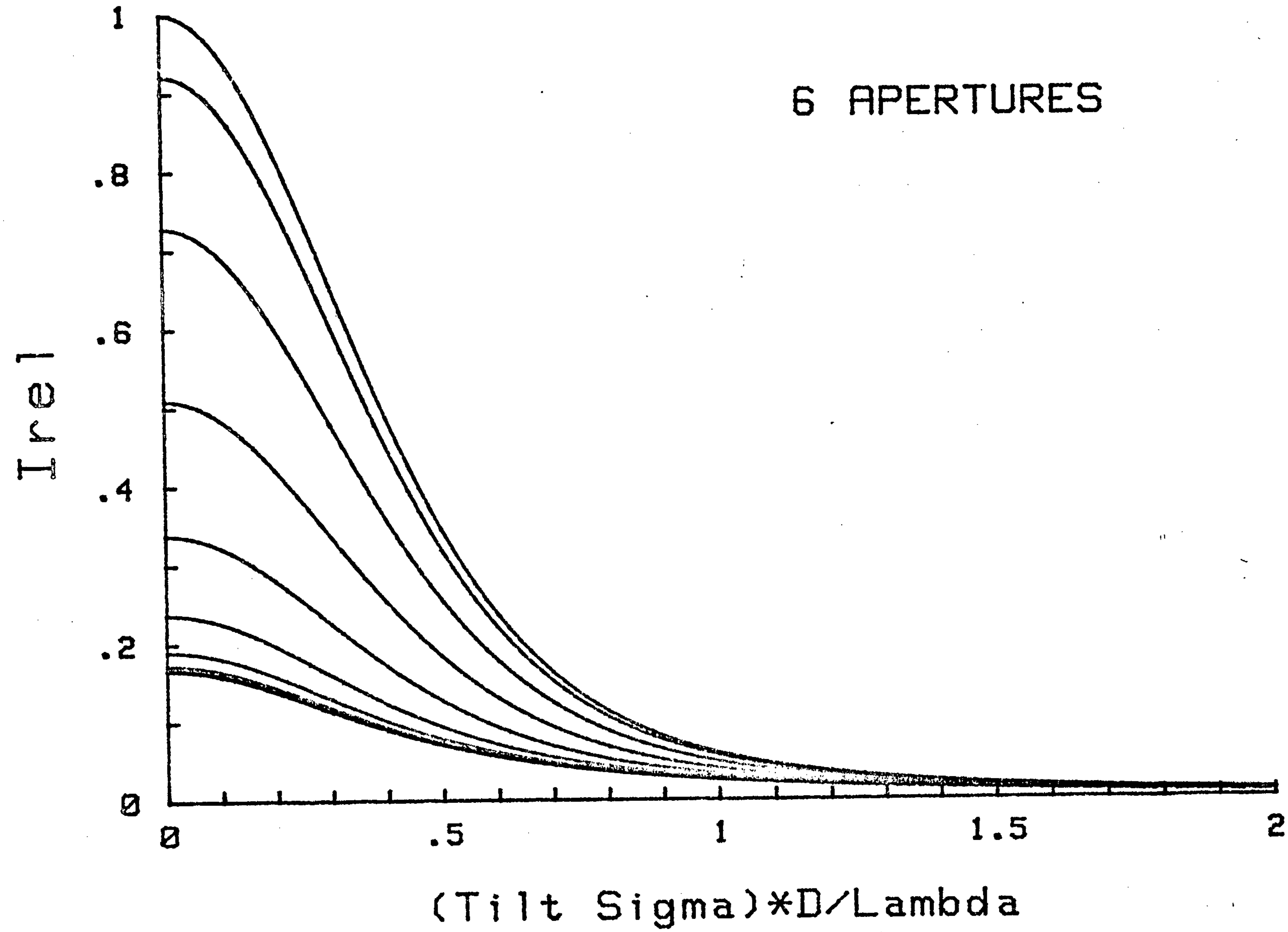
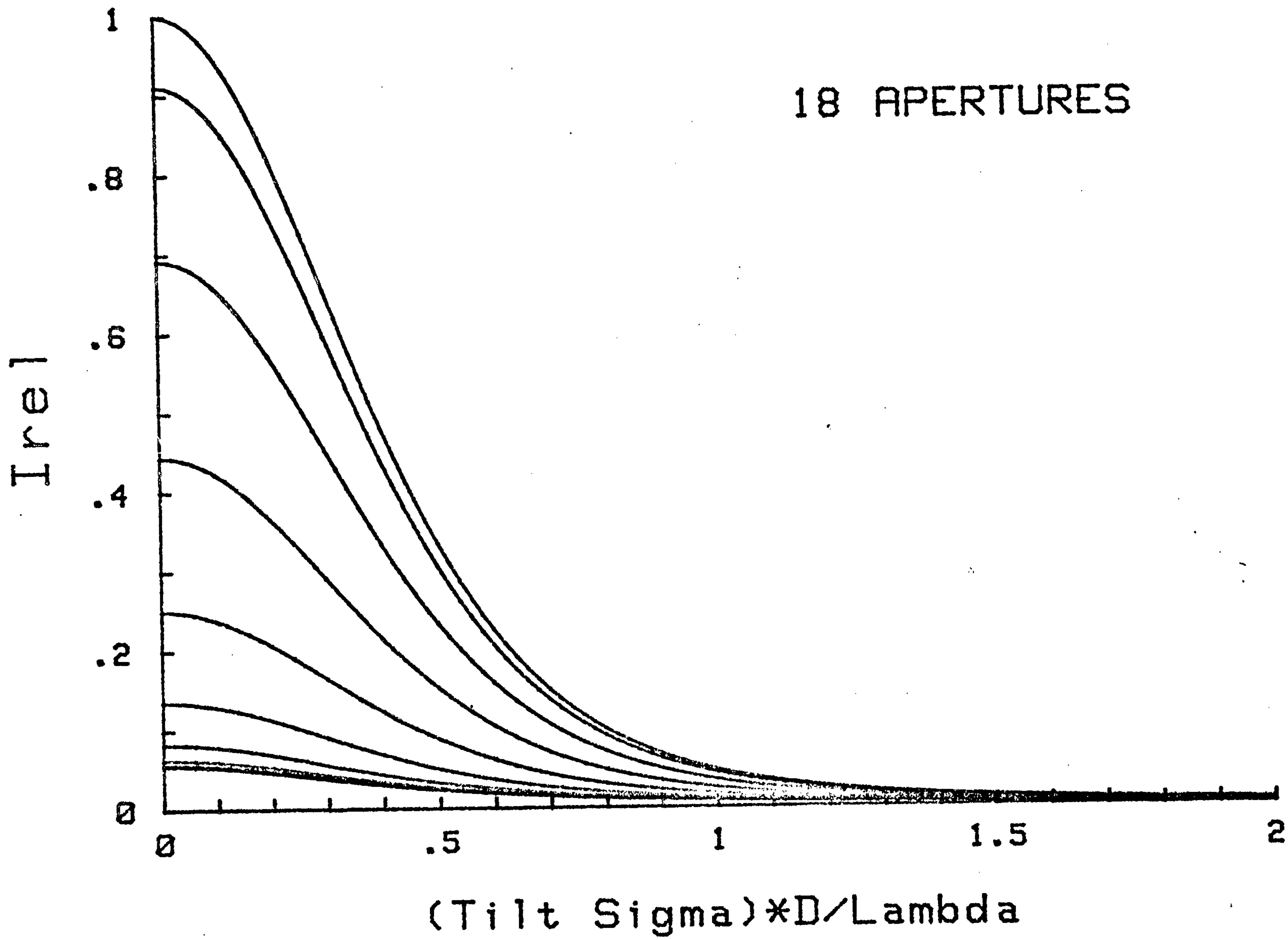
2. M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions (National Bureau of Standards, 1964).


1 APERTURE
Irel
1
.8
.6
.4
.2
0
0
.5
1
1.5
2
(Tilt Sigma)*D/Lambda


6 APERTURES
Irel
1
.8
.6
.4
.2
0
0
.5
1
1.5
2
(Tilt Sigma)*D/Lambda


18 APERTURES
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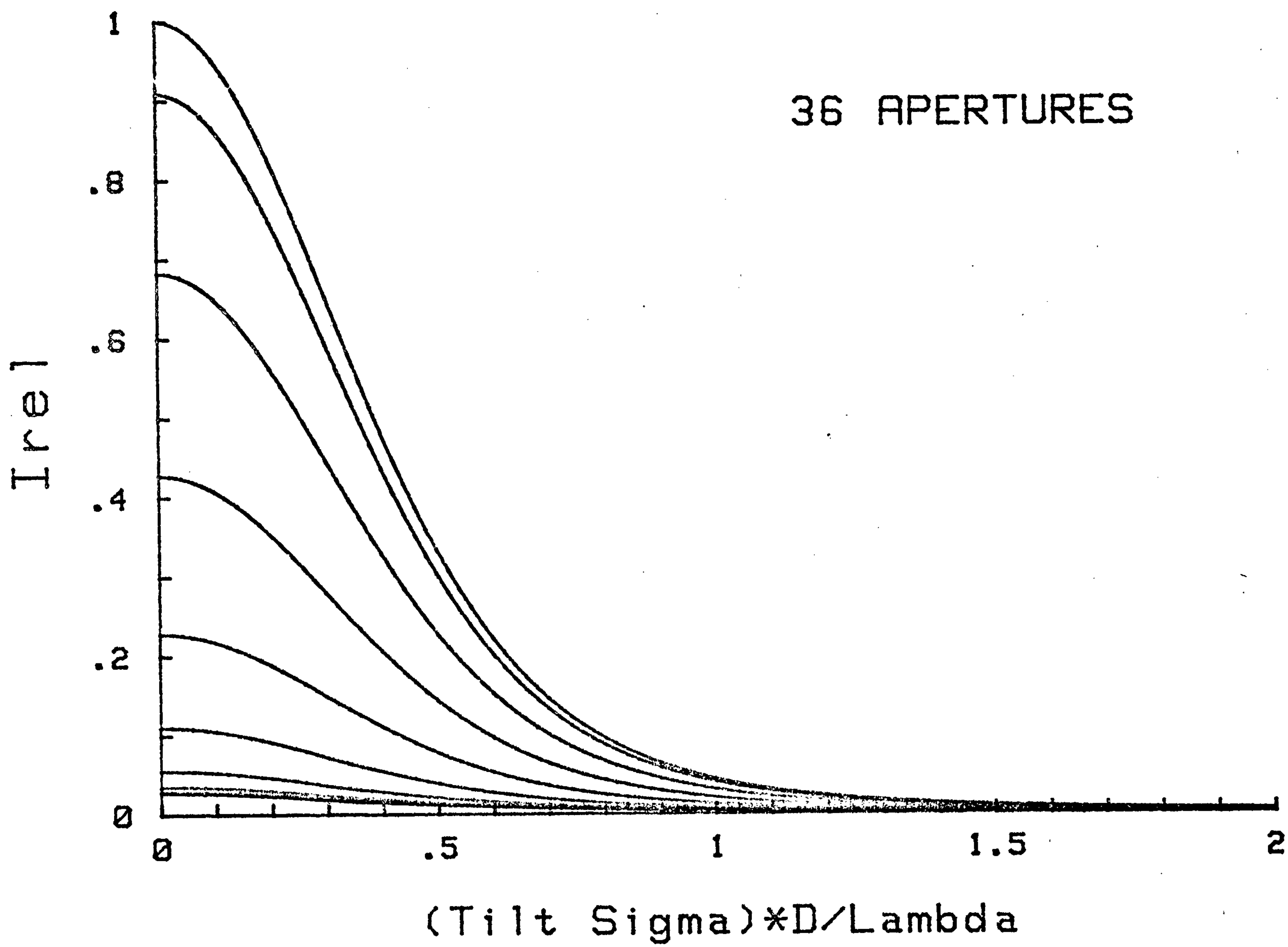
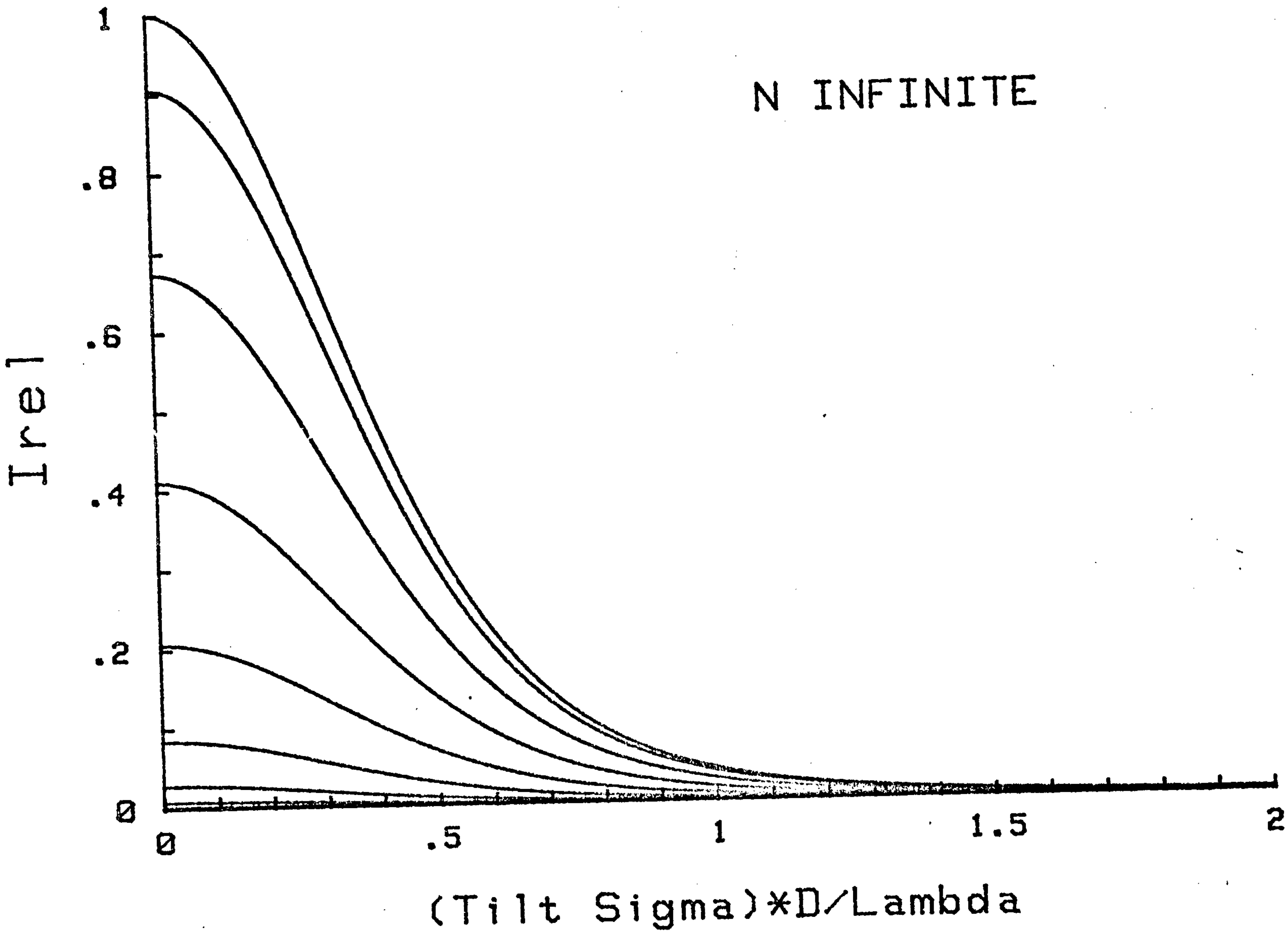
.8
.6
.4
.2
0
Irel
0
.5
1
1.5
2
(Tilt Sigma)*D/Lambda



36 APERTURES
Irel
1
.8
.6
.4
.2
0
0
.5
1
1.5
2
(Tilt Sigma)*D/Lambda


N INFINITE
Irel
1
.8
.6
.4
.2
0
0
.5
1
1.5
2
(Tilt Sigma)*D/Lambda